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Trapped Flux in Superconductors

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TRAPPED FLUX IN SUPERCONDUCTORS

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CONTENTS

When a magnetic field is applied to a superconductor the normal state may be restored, and on removing the field the superconducting state is re-established, usually with a proportion of the field trapped in normal channels. The amount of flux trapped has been studied systematically as a function of temperature in rods of pure tin and of tin alloyed with indium up to 3%. In order to obtain significant results the specimens must be single crystals, homogenized by prolonged annealing, and having well-polished surfaces. The proportion of flux trapped is very small (~0·1%) in pure tin, increasing steadily as the indium concentration is increased. For indium concentrations less than about 2·3% the proportion trapped tends to zero as the temperature tends to the transition temperature. For greater indium concentrations there is a sharp rise in trapping to a very high value (~50%) at the transition temperature. The trapped flux is rather firmly bound.

In order to account for these results a model of the superconducting state is developed, based on the theories of London & London and of Gorter & Casimir, and incorporating the idea of coherence. Typical processes such as spontaneous nucleation of the superconducting phase are analyzed and used to discuss the factors influencing the coalescence of adjacent superconducting domains, which is an essential part of the trapping mechanism. It is concluded that for not too great indium concentrations coalescence is achieved only through the presence of flaws, and that the sudden change in behaviour at 2·3% indium marks the beginning of spontaneous coalescence. The model appears to be capable of accounting qualitatively for most of the details of the observed behaviour.

1. Introduction

It was established by the classical experiments of Meissner & Ochsenfeld (1933) that the superconducting state is one of virtually perfect diamagnetism, magnetic field being completely excluded except from a very thin penetration layer. If a superconductor is placed

in a sufficiently strong field, the field enters as the specimen makes a transition to the normal state; when the field is reduced again the specimen returns to the superconducting state with ejection of the flux. The return to superconductivity is not abrupt if the body has a non-vanishing demagnetizing factor, but takes place by way of the intermediate state, a finely divided mixture of normal and superconducting regions. As the field is lowered the proportion of superconducting material increases until eventually, under ideal conditions, all the normal regions, through which the flux runs, are eliminated and the body is wholly superconducting. In practice the ideal behaviour is never quite realized, except perhaps in very small bodies,* and the specimen after removal of the external field possesses a small residual magnetic moment due to flux which is trapped in normal channels through the otherwise superconducting specimen. On account of the perfect conductivity of the walls of the channel the flux cannot be removed except by the migration of the channel to the surface of the specimen.

In other investigations of the magnetic properties of superconductors there have often been observations of the magnitude of the trapped flux, but these have usually been incidental and quite unsystematic. An exception was the work of Stout & Guttman (1952) on alloys of indium with thallium, in which considerable attention was paid to flux-trapping and its variation with the composition of the alloy. But even here, largely because several other aspects of the magnetic behaviour were studied in detail, the amount of information obtained on this particular point was insufficient to allow the recognition of any general features of the phenomenon, beyond the fact that those alloys which contained more than about 10% of thallium were notably efficient as trappers of flux. The intention of the present work was to concentrate solely on this one aspect and to see whether a detailed study of a range of alloys would reveal any regularities of behaviour. Although such an investigation may be regarded as having a certain interest in its own right, this was not the primary motive of the experiments. In order that flux may be trapped it is necessary that adjacent superconducting regions in the intermediate state shall be able to coalesce so as to enclose normal regions and prevent the elimination of the flux which they carry. Thus a study of flux-trapping may result in a better understanding of phenomena in the neighbourhood of the phase boundary. In particular, the concept of 'coherence' (Pippard 1953*b*) leads to the suggestion that in pure materials and dilute alloys the coalescence of superconducting regions should be hard to achieve, but that no such barrier to coalescence should exist in alloys containing more than a certain critical concentration of solute. The experiments were designed especially to study this point, by finding whether there was such a critical concentration above which the amount of trapped flux rose sharply, in the belief that in this way light might be thrown on a rather fundamental aspect of the nature of the superconducting state. As will be seen, something like the expected behaviour was in fact revealed, although the hope of a clear-cut confirmation of the underlying ideas proved to have been too naïve. At the same time the flux-trapping has been found to exhibit such marked regularities of behaviour that it

* The quantization of flux (London 1950) in units of hc/e, i.e. 4×10^{-7} G cm^2, should ensure that no flux is trapped in colloidal particles of diameter less than about 10^{-4} cm, and may well play an important role in bodies of somewhat larger dimensions. It should not, however, be significant in the large specimens used here.

seems worth while to attempt to account for them in terms of an idealized model of a superconductor. It is too much to suppose that anything approaching a complete explanation can be given of this undoubtedly complex phenomenon, to which a great many diverse factors contribute, and many of the suggestions which will be put forward must be regarded as speculative. Nevertheless, the degree of success which is attainable by the introduction of only a few, and not unreasonable, hypotheses encourages the belief that the idealized model chosen, and in particular the coherence concept which it incorporates, does indeed represent something of the basic nature of a superconductor.

It should be made clear at the outset that the experiments were aimed at elucidating the flux-trapping properties of homogeneous superconductors only, and that pains were taken to avoid as far as possible both physical and chemical inhomogeneities such as are known (Mendelssohn 1935) to give rise to extensive trapping. For this reason the alloys studied were dilute solutions (up to 3%) of indium in tin, since there is good evidence that this system forms a substitutional solution in concentrations at least up to 5%, and there should therefore arise no extraneous effects from the precipitation of other alloy phases. For this reason also the specimens were single crystals, and other precautions to achieve homogeneity were taken, as will be described in due course.

2. Experimental method

In order to simplify comparison between results obtained with different specimens it was decided to work with one standard form of specimen, a long circular rod, which was to be subjected to a transverse magnetic field. This arrangement, besides being convenient, ensured that as the magnetic field was removed the specimen passed through the intermediate state in the range of field $H_c > H > \frac{1}{2}H_c$ (H_c being the critical field), so that every opportunity was given for superconducting regions to coalesce into rings and thereby trap flux. To avoid possible extraneous effects arising near the cut ends of the rods the magnetic moment due to trapped flux was measured only over the central region of the specimen, by means of the apparatus shown diagrammatically in figure 1. The specimen holder A was a Paxolin tube having its lower end B removable for insertion of the specimen. At the upper end the tube was cut away for a short length to leave two vertical guides C in which ran a Paxolin collar D; the upper end of the specimen E was cemented into D, so that it was prevented from rotating but was nevertheless free enough not to be strained by differential contraction of the specimen and its holder. The specimen holder was free to rotate in a copper-nickel tube F, alinement being maintained by means of the bearings G and H. A thin tube J enabled the rotation to be effected from outside the cryostat. A number of holes K allowed access of liquid helium to the specimen. Two parallel coils L of about 20 000 turns each, having their axis horizontal, were mounted on the tube F, and were connected in series to a galvanometer. Outside the cryostat a large Helmholtz pair, with axis parallel to the axis of L, provided a uniform field to destroy superconductivity in the specimen. The procedure in determining trapped flux was to switch on a field greater than H_c and remove it again; the specimen holder was then rotated sharply through 180° and the galvanometer throw recorded. In order to relate this to the magnetic moment per unit length of the specimen, fine strips of copper were

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attached along the length of each side of the specimen holder, and joined at the lower end. This loop provided, when a known current was passed through it, a standard magnetic moment to calibrate the apparatus. It also enabled the transition temperature, T_c, of the specimen to be determined easily; the mutual inductance between the loop and the coils L differed appreciably according as the specimen was normal or superconducting, so that by observing the galvanometer throw when a small current was switched into the loop the transition to the superconducting state was immediately perceptible. The sensitivity of the apparatus was such that a magnetic moment of 5×10^{-5} e.m.u./cm of rod could just be detected. For a specimen of diameter 4 mm such a moment corresponds to an average trapped field strength of 5×10^{-3}G. It was thus possible to work so near the transition

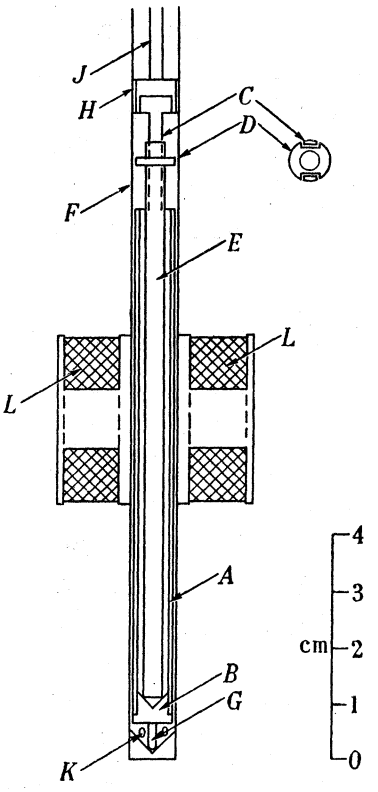

FIGURE 1. Apparatus for measuring trapped flux.

temperature that H_c was only a few Gauss and still detect the effect of less than 1% flux-trapping. The fraction of flux trapped is defined as $4\pi I/H_c$, in which I is the magnetic moment per unit volume after the field is removed. If all the flux contained in the specimen at the point in the magnetic cycle when its magnetic moment reached a maximum value ($H_c/4\pi$ according to the simple theory of the intermediate state) were then trapped, the fraction trapped would be unity with this definition.

The specimens, whose properties are listed in table 1, were single crystal rods 10 cm long and usually between 3 and 4 mm in diameter. They were prepared by casting in glass tubes and allowing cooling to proceed slowly from one end; a narrow constriction near the cooler end helped in the production of a single crystal, but it was difficult to obtain single crystals of this size with an indium concentration greater than about 2·5%,

and rather smaller diameters were used for the least pure specimens. After cooling, the glass was dissolved in hydrofluoric acid. As a result of the discovery by Doidge (1954) that the sharpness of the transition of a tin-indium alloy is greatly improved by annealing, all the specimens, except those of pure tin, were annealed *in vacuo* at a temperature of about 205° for a period not less than 18 days, and often considerably longer. The effect of annealing was investigated with a specimen containing 1% indium, and the results will be presented in due course. It was believed during the greater part of the experimental program that this method of preparation resulted in the production of crystals as nearly

TABLE 1

specimen no.	nominal % indium by weight	estimated* % indium by weight	diameter (mm)	orientation† of tetrad axis	annealing time (days)
1	0	0‡	3·84	86	4
2	0	0‡	3·82	82	4
3	0·20	0·20	4·06	85	32
4	0·50	0·52	3·13	88	19
5	1·00	1·02	3·14	88	19
6	1·00	1·03	3·18	77	19
7	1·00	1·02	2·97	83	various§
8	1·80	1·80	3·88	90	18
9	1·90	1·94	3·79	87	25
10	2·00	1·98	3·63	87	19
11	2·00	1·94	3·13	84	19
12	2·30	2·26	3·65	74	23
13	2·30	2·38	3·74	86	23
14	2·30	2·33	3·76	86	34‖
15	2·50	2·46	3·74	86	32
16	2·50	2·45	2·60	76	110
17	2·70	2·65	3·17	84	20
18	3·05	2·94	2·22	80	24
19	1·00	—	3·75	polycrystal	50
20	2·70	2·60	—	86	21
21	3·05	2·94	—	86	21
22	0·84% Zn	—	3·43	87	55

* Calculated from residual resistivity; see text.
† Angle between tetrad axis and axis of specimen.
‡ Specpure specimens having residual resistance too small to be measured.
§ Used for studying effect of annealing; see text.
‖ Subsequently annealed a further 90 days.

perfect as could be obtained, but this unfortunately is not quite true. It was early observed that if the specimens were etched in hydrochloric acid before annealing they tended to etch preferentially so as to exhibit ridges running about parallel to their length, but it was not until nearly all the measurements were made that the significance of this was realized. It is almost certain that during the cooling of the specimens to room temperature after crystallization the differential contraction of glass and tin causes a small amount of slip, and that it is the slip planes which are shown up by etching. The prolonged annealing eliminates most of the deformations resulting from slip, so that preferential etching is no longer apparent, but, as the results will show, there is still a residual deformation which may influence the behaviour. In order to test the importance of this factor two specimens were crystallized in the open, on flat glass plates. Although now (it was hoped) free from

slip they were in other ways not entirely satisfactory, being somewhat flattened in cross-section and anisotropic in their flux-trapping. Nevertheless, they served to indicate that the crystalline distortion due to slip, which remained after prolonged annealing, produced a less serious effect than might have been feared.

After the measurements on flux-trapping were complete the residual resistance of each specimen was compared with its resistance at 0° C, and the orientation of each crystal was determined. It is interesting to note (see table 1) how strong is the preference for growth in a direction normal to the tetrad axis; in fact, the crystallographic examination reveals an even more specific preference for growth normal to one of the (110) planes of the tetragonal lattice, while the slip planes shown up by etching are parallel to the other (110) plane. This explains why the observed planes nearly always ran within 5° of the axis of the specimen.

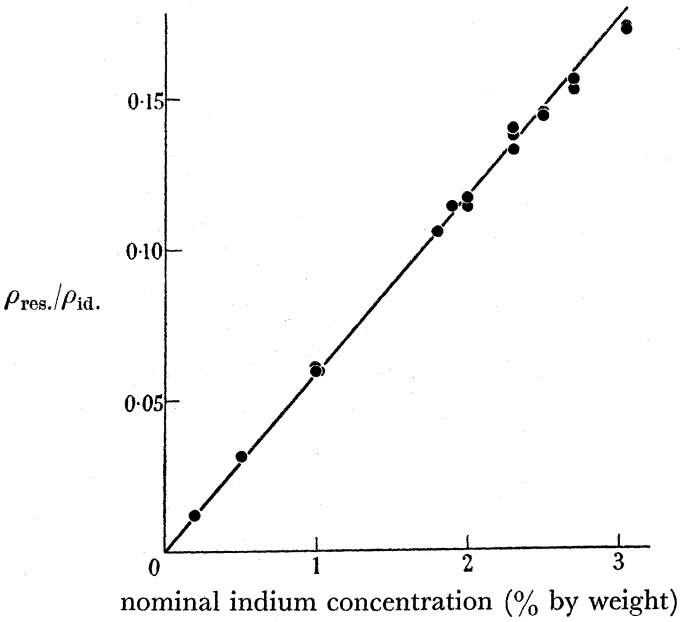

FIGURE 2. Residual resistivity of tin-indium alloys.

The purpose of measuring the residual resistance was to obtain a measure of the impurity content more reliable than the nominal indium content (although, as it turns out, the latter would be good enough for most purposes), and also because probably what matters basically in the present problem is not the amount of indium, but the mean free path of the conduction electrons, which is directly related to the resistance. To obtain the residual resistivity from the measured ratio $R_{\text{res.}}/R_{0°\,\text{C}}$ it was necessary in the least pure specimens to make some allowance for the fact that the specific resistance at room temperature is not the same as in pure tin. From absolute measurements on three specimens it was concluded that Matthiessen's rule was not exactly true, and that a more satisfactory rule for the present purpose would be the following:

$$R_{0°\,\text{C}} = R_{\text{id.}} + 1{\cdot}12R_{\text{res.}},$$

where $R_{\text{id.}}$ is the ideal resistance at 0° C. It was also necessary to allow for a difference in the anisotropy of the residual and ideal resistivities (Bridgman 1925; Pippard 1953*a*).

These factors are taken into account in the formula used in reducing the resistance observations:

$$\frac{\rho_{\text{res.}}}{\rho_{\text{id.}}} = (1+0{\cdot}30\cos^2\theta)\frac{r}{1-1{\cdot}12r},$$

in which $\rho_{\text{res.}}$ and $\rho_{\text{id.}}$ are respectively the residual resistivity and the ideal resistivity at 0° C, for current flow normal to the tetrad axis, θ is the angle between the tetrad axis and the axis of the specimen, and r is the measured ratio $R_{\text{res.}}/R_{0^\circ\text{C}}$. If each indium atom acted independently of the rest as a scatterer we should expect $\rho_{\text{res.}}/\rho_{\text{id.}}$ to be proportional to indium concentration, and it will be seen from figure 2 that agreement is good except for the highest concentrations. It is most likely that here the departure from the straight line is simply due to a slight segregation of indium during crystallization. In table 1 what is called the estimated indium concentration is taken from the straight line of figure 2, that is, on the assumption the $\rho_{\text{res.}}/\rho_{\text{id.}} = 5{\cdot}88f$, f being the proportion by weight of indium in the alloy.

3. Experimental results

3·1. *Presentation*

The results obtained will be expressed in graphical form by plotting the fraction of trapped flux α $(=4\pi I/H_c)$ as a function of reduced temperature t $(=T/T_c)$ for each specimen. Reduced temperatures are used in order to facilitate comparison between specimens of different indium content, as the transition temperature T_c varies slightly with concentration of indium, from 3·73° K for pure tin to 3·63° K for tin+3% indium. For the calculation of α it has been assumed that the critical field curves for different alloys are geometrically similar, i.e. $H_c/H_0 = F(T/T_c)$, where H_0 is the critical field at 0° K, and $H_0/T_c =$ constant. The form of $F(T/T_c)$ used is that given by Lock, Pippard & Shoenberg (1951) for pure tin. It is unlikely that any noticeable error arises from this assumption, but if future measurements should reveal a significant variation with indium content of the shape of the critical field curve the results given here can easily be corrected.

A number of experiments were performed to investigate the effect of various types of imperfection in the specimens, surface roughness, polycrystallinity and inhomogeneity of impurity. Each type of imperfection was found to produce its characteristic effect.

3·2. *Surface roughness*

Specimen 2 was etched after crystallization and before annealing, and preferential etching along slip planes gave it a laminated appearance, two quadrants being terraced on a scale visible to the naked eye, while the other two quadrants were etched without visible structure. A cross-section of the specimen would have had the form shown schematically in figure 3. The trapped flux was studied as a function of the disposition of the terraces relative to the field, and figure 4 shows the behaviour at a temperature of 1·77° K. In the same diagram is shown the behaviour after electropolishing of the specimen and removal of the terraces. While before polishing the value of α varied between 15 and 0·35%, afterwards it was nearly constant at 0·22%. Since it is at that part of the surface which is more or less parallel to H that the field is greatest, and here that any disturbance caused

by surface roughness will have most effect, we must ascribe the peak of flux-trapping at values of ϕ around 90° to the macroscopic roughness of the terraces. The effect of microscopic etch pits, at $\phi \sim 0°$, is much less marked, but it is nevertheless desirable to avoid any suspicion that the results are influenced by surface topography, and for this reason all the specimens were electropolished. Small anisotropies of α were still observed, but were usually less than $\pm 5\%$ in relative amplitude and unsystematic in their variation from one specimen to another. They were not studied in detail, and measurements were usually taken at that specimen orientation for which α had the lowest value. In one or two specimens of high indium content it was noticed that the small variations of α with ϕ were temperature-dependent, and that the positions of the maxima and minima of α might be interchanged as the temperature was lowered. This suggests that part at any rate of the observed anisotropy is intrinsic to the material and not an extraneous topographical feature.

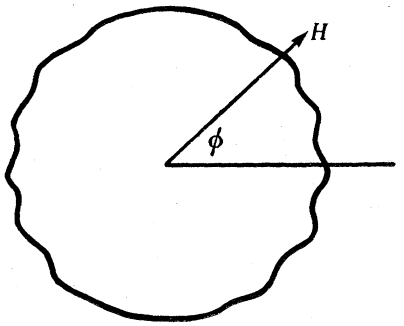

FIGURE 3. Schematic cross-section of etched specimen.

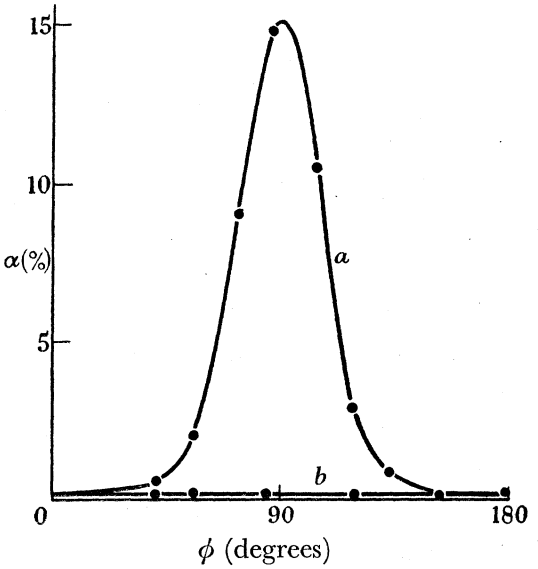

FIGURE 4. Variation of trapped flux with orientation for (*a*) etched specimen and (*b*) the same specimen after electropolishing.

3·3. *Polycrystallinity*

A polycrystalline specimen (sp. 19) containing 1% indium was annealed for 50 days, after which it was found that the crystallites were several millimetres in size, many of them extending over the whole cross-section of the specimen. It was electropolished and measured, the variation of α with t being as in figure 5. Also shown is the curve for sp. 6, a single crystal of the same composition. It appears that the crystal boundaries seriously affect the behaviour only near T_c, but as the curve for sp. 19 closely resembles that for the least pure specimens it is important to use only single crystals if confusion of the effects of crystal boundaries and of impurities is to be avoided. It will be observed that the values of α for sp. 6 have been scaled down so as to show the similarity of the curves at low temperatures. The justification for this procedure will be discussed later.

3·4. *Effect of annealing*

Specimen 7, containing 1% indium, was electropolished after crystallization and measured unannealed, and then after 44 h and a further 42 days' annealing. The improvement in performance is shown in figure 6. It is probable that the unannealed specimen was somewhat strained by slip, and also inhomogeneous in its composition, owing to local segregation of indium during crystallization. At all events, annealing is an essential part of the preparation of good alloy specimens, and the annealing must be fairly thorough, to allow time for diffusion of indium into a uniform concentration and for the majority of the strains to be eliminated. To judge from the results obtained with the other specimens, 5 and 6, of the same composition, 19 days' annealing was sufficient to eliminate the sharp rise in α towards T_c, and once this has gone very little further improvement is effected by more prolonged annealing. As we shall see, however, with sp. 14 there was a significant change between 34 and 124 days' annealing, so that the process of homogenization must be extremely slow.

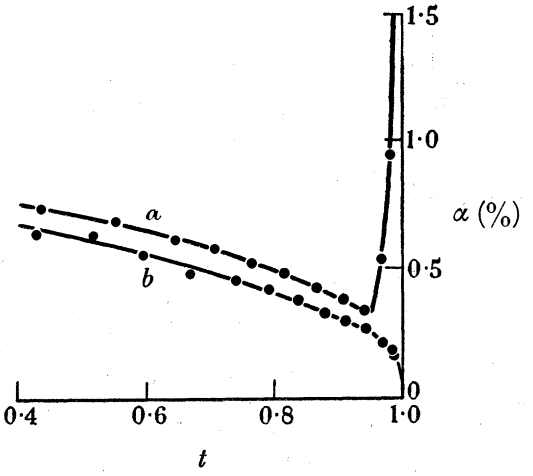

FIGURE 5. Trapped flux in Sn + 1% In: (*a*) polycrystalline specimen 19 (α as measured); (*b*) monocrystalline specimen 6 ($\alpha \times 0{\cdot}39$).

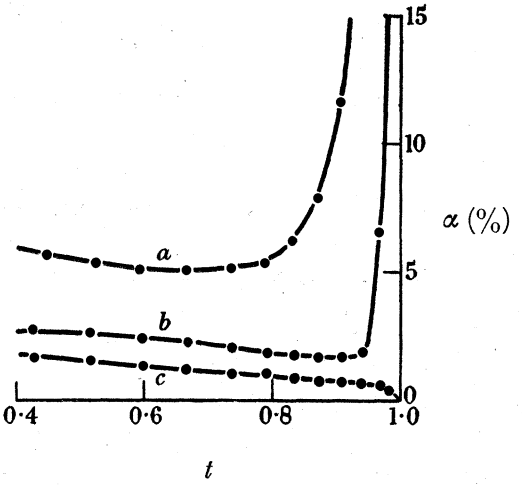

FIGURE 6. Effect of annealing on sp. 7: (*a*) unannealed; (*b*) annealed for 44 h; (*c*) annealed for 44 days.

3·5. *Insoluble impurities*

Zinc is soluble in tin only to the extent of 0·4% (Serin & Lohman 1953), so that sp. 22, which contained 0·84% zinc, must have been inhomogeneous, with inclusions of nearly pure, non-superconducting zinc. The trapped flux here follows a quite different temperature variation (figure 7), and is at all temperatures of large magnitude. It is to be presumed that this specimen behaves like a 'Mendelssohn sponge' (1935), the normal inclusions acting as traps for flux, and as barriers to the free expansion of superconducting domains.

3·6. *Results*

In the specimens which were actually used for comparison of the effects of different impurity contents these faults were avoided as far as possible, by employing only well-annealed, polished single-crystal specimens, and the consistency of the results obtained

suggests that they represent the behaviour of homogeneous superconductors, differing significantly only in the mean free paths of the conduction electrons, and in the density and nature of those lattice defects which are probably unavoidable even in the most carefully prepared crystals. The quality of the specimens is indicated by the fact that they all had extremely sharp magnetic transitions, measured as described above, changing from completely normal to completely superconducting in a temperature range of usually less than 0·004° K. The consistency of behaviour may be treated under three headings: reproducibility of α in successive magnetic cycles under identical conditions; consistency between specimens having the same indium concentration; and systematic variation of properties with indium concentration.

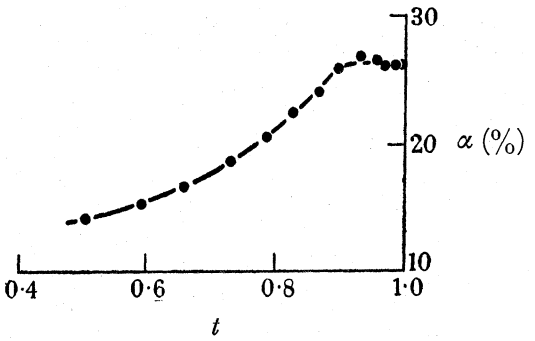

FIGURE 7. Trapped flux in sp. 22 (Sn + 0·84 % Zn).

It was very rare for the amount of trapped flux to vary by more than 5 % in a repetition of the magnetic cycle, which consisted of the application and removal of a field greater than H_c, and often the agreement between successive cycles was better than 1 %. The same degree of reproducibility may be observed in the way in which the experimental points lie close to a smooth curve when α is plotted against t. The only way in which inconsistency might be achieved was by altering the speed with which the field was removed. For example, in pure tin α was approximately doubled, at values of t greater than 0·8, if the field was switched off suddenly rather than gradually lowered; at lower temperatures the effect was not so marked, amounting to a difference of only 20 %. In less pure tin the difference was much less marked and was often unobservable. This may partly be a consequence of the contraction of the time scale in the specimens of lower conductivity; one would expect the speed of all processes limited by eddy currents to vary as $\sigma^{-\frac{1}{2}}$, so that the time scale in tin + 1 % indium should be about 17 times faster than in pure tin. It may therefore be that the most rapid removal of field was still slow compared to the rate at which reorganization of the intermediate state could occur in the alloys, but this is not the whole explanation. For with indium concentrations of 1 % or over it was often found that α was slightly greater (by no more than 2 % usually) when the field was reduced slowly. This particular aspect of the problem has not been studied intensively, and the rate of reduction of the field was chosen for each specimen to give the lowest value of α.

The consistency of behaviour between specimens having the same indium concentration did not lie in their showing identical values of α at a given temperature, for there might be as much as a factor of 2 between different specimens, but in the fact that the shapes of the

curves relating α to t were similar for different specimens. Examples are shown in figures 8, 9 and 10, the measured values of α being scaled by the amounts indicated in the captions. The accuracy of the points for pure tin (figure 8) is not high because of the very small fraction of flux trapped, but even so it can be seen that the two specimens give nearly the same curves in spite of a scale difference of 2·25. The other figures make it clear that the consistency is by no means perfect. Nevertheless, it seems reasonable to conclude that the shape of the α-t curve is more significant than the exact scale, which may be determined by factors which vary from one specimen to another.

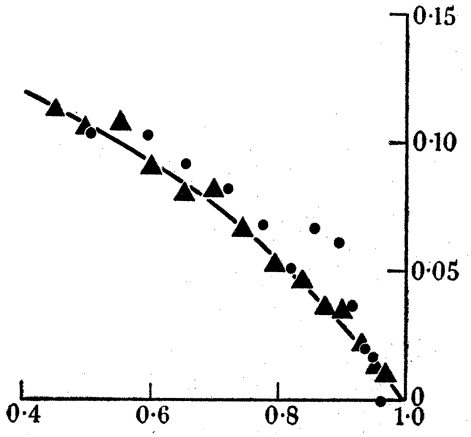

FIGURE 8. Trapped flux in pure tin: • sp. 1 (α as measured); ▲ sp. 2 ($\alpha \times 0{\cdot}44$).

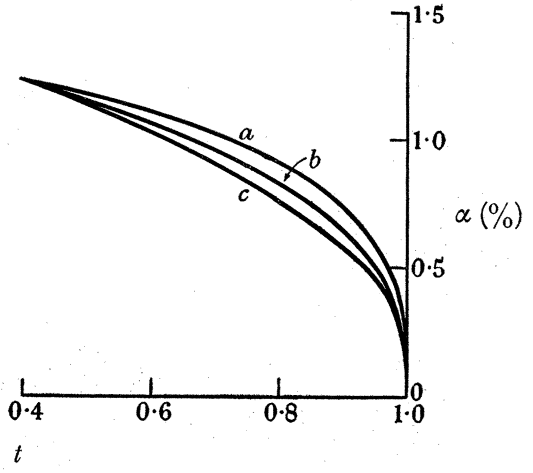

FIGURE 9. Trapped flux in Sn + 1% In: (*a*) sp. 5 ($\alpha \times 0{\cdot}82$); (*b*) sp. 6 (α as measured); (*c*) sp. 7 ($\alpha \times 0{\cdot}72$).

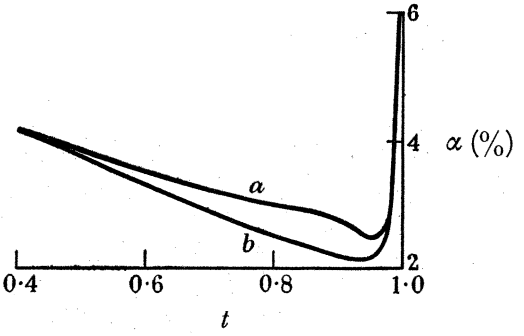

FIGURE 10. Trapped flux in Sn + 2·3 % In: (*a*) sp. 14 (α as measured); (*b*) sp. 13 ($\alpha \times 0{\cdot}66$).

In view of this it is probably not a falsification of the results to scale the values of α by a suitable factor for each specimen, as in figure 11, so as to exhibit more clearly the systematic change in shape which takes place as the indium concentration is increased. The scaling factors used are given in the legend; they have been chosen so as to make the curves lie evenly between those for sp. 1 (pure tin) and sp. 18 (2·94 % indium). For pure tin, as is seen more clearly in figure 8, α falls nearly linearly to zero as t approaches unity, but as the indium concentration is increased the fall to zero becomes progressively sharper, until at about 1·9 % a totally different behaviour sets in; instead of tending to zero at the

transition temperature α rises sharply to 0·5 or more. The limiting value is hard to determine with certainty, since in the restricted temperature range just below T_c the values assumed for H_c depend critically on the value chosen for T_c, and an error of 10^{-3}° K makes a substantial difference to the limiting value of α. For all specimens exhibiting this type of curve the rate of rise towards T_c is so great that there is a temperature range in which not only α, the proportion of trapped flux, but the amount of trapped flux itself increases as H_c falls. The largest values of α observed were with sp. 13, and are shown on an extended scale in figure 12. By the time the indium concentration has reached 2% the low-temperature end of the curve has become nearly level, but with higher concentrations it

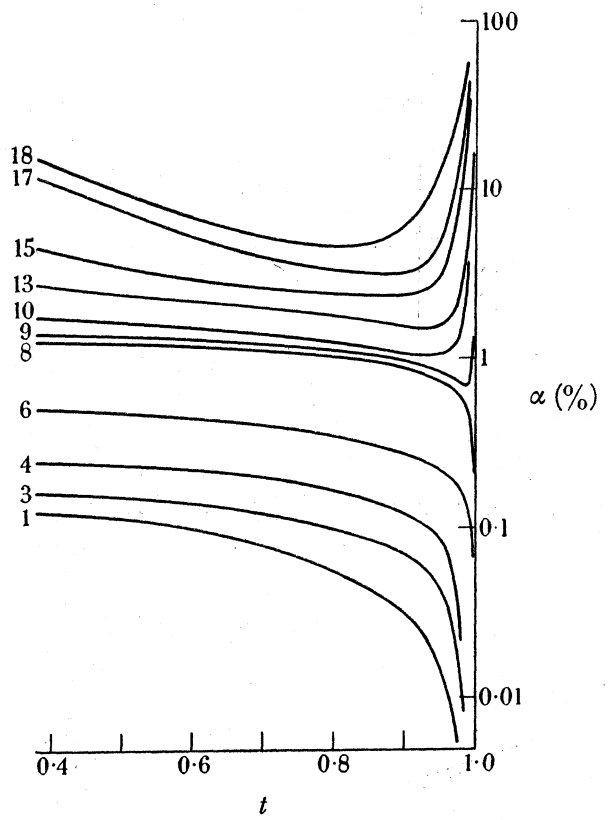

FIGURE 11. Collected results on trapped flux (α plotted logarithmically). The serial numbers of the specimens are shown to the left.

specimen	1	3	4	6	8	9	10	13	15	17	18
scale factor	1·00	0·21	0·40	0·41	0·45	0·46	0·80	0·48	1·14	1·00	1·00

begins to show a tendency to rise rather abruptly, and this tendency is fully developed in sp. 18. It is unfortunate that failure to grow single crystals of the necessary size with more than 3% indium has prevented the continuation of this process from being followed. One would like to know whether the minimum persists in really impure homogeneous superconductors. Before leaving this general description of figure 11 it should be noted that the anomalous shape of the curve for sp. 10 is probably due to an accident which led to a severe jarring of the specimen as it was being removed from the annealing oven. Although it was re-annealed for a short time it seems that the damage it suffered was not completely eliminated.

After this series of measurements was concluded, with such apparently satisfactory results, sp. 14 was remeasured, having been annealed for a further 90 days. It was disturbing to find that the rise of α towards T_c had completely disappeared, as shown in figure 13. Apart from this, and a general reduction in scale by a factor 1·5, there was no significant change in the shape of the curve. It appears then that the crystal deformations resulting from the method of preparation of the specimen take an exceedingly long time to be eliminated by annealing, and play an important part in trapping flux in the neighbourhood of the critical temperature, though not at lower temperatures, except as a factor determining the scale of α.

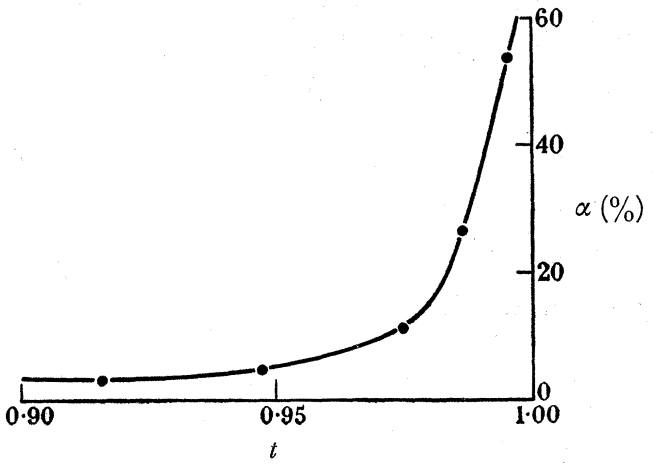

FIGURE 12. Trapped flux in sp. 13 near transition temperature.

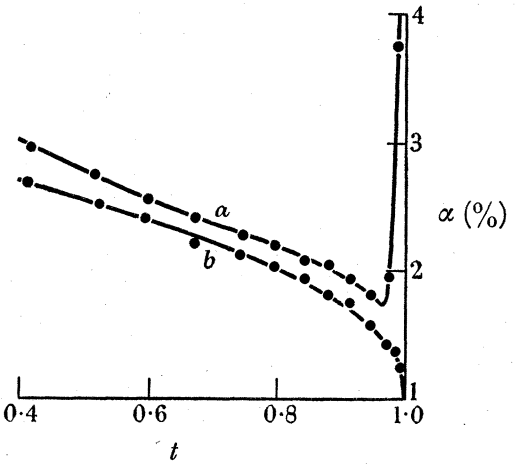

FIGURE 13. Effect of prolonged annealing on behaviour of sp. 14: (*a*) annealed 34 days; (*b*) annealed 124 days.

This experiment with sp. 14 does not imply, however, that the rapid rise near T_c is entirely to be ascribed to crystal defects, since it was still present in sp. 16 which, though grown in a tube and therefore probably strained, was annealed for 110 days, nearly as long as sp. 14. The same rise was also shown by sp. 20 and 21 which were grown in the open and not subjected to comparable stresses on cooling. The results for these specimens are exhibited in figure 14. The absence of experimental points on the curves for sp. 20 and 21 arises from the flattened form of the specimens, which led to a highly anisotropic behaviour of α; the curves shown are the mean of maximum and minimum curves, and should be comparable with those for a circular specimen. What is notable about these curves, apart from the rapid rise, is that the rise at low temperatures, though still present, is far less marked than in the curves for comparable specimens in figure 11. We may conclude from this that the rise at both ends is a characteristic property of unstrained impure specimens with indium concentrations greater than about 2·5 %, and that strains enhance this property and cause it to appear at rather lower concentrations. Otherwise it does not seem (see figure 13) that residual strains play any great part in determining the shape of the α-t curve.

Finally, a few, rather unsystematic, experiments must be described which were aimed at determining how firmly the trapped flux was held inside the specimen. The specimen

was subjected to the usual cycle of applying and removing a field greater than H_c, and was then rotated through 180°. A small field was now applied in the same direction as before, so that it opposed the flux trapped in the specimen. Since the lines of force of the trapped flux run in the reverse direction around the outside of the rod, the applied field tends to increase

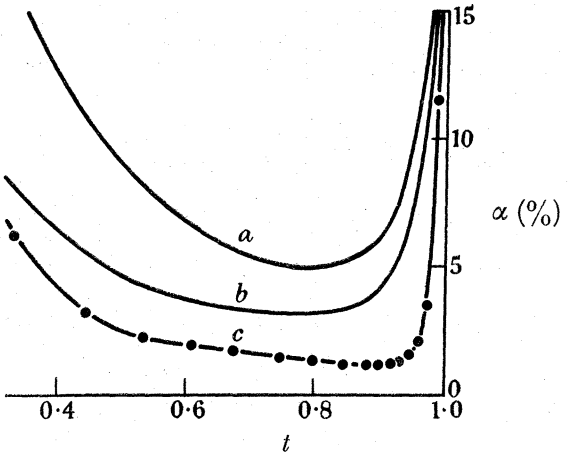

FIGURE 14. Trapped flux in alloys containing more than 2·4 % indium: (*a*) sp. 21 (2·94 %); (*b*) sp. 20 (2·60 %); (*c*) sp. 16 (2·45 %). All values of α are unscaled.

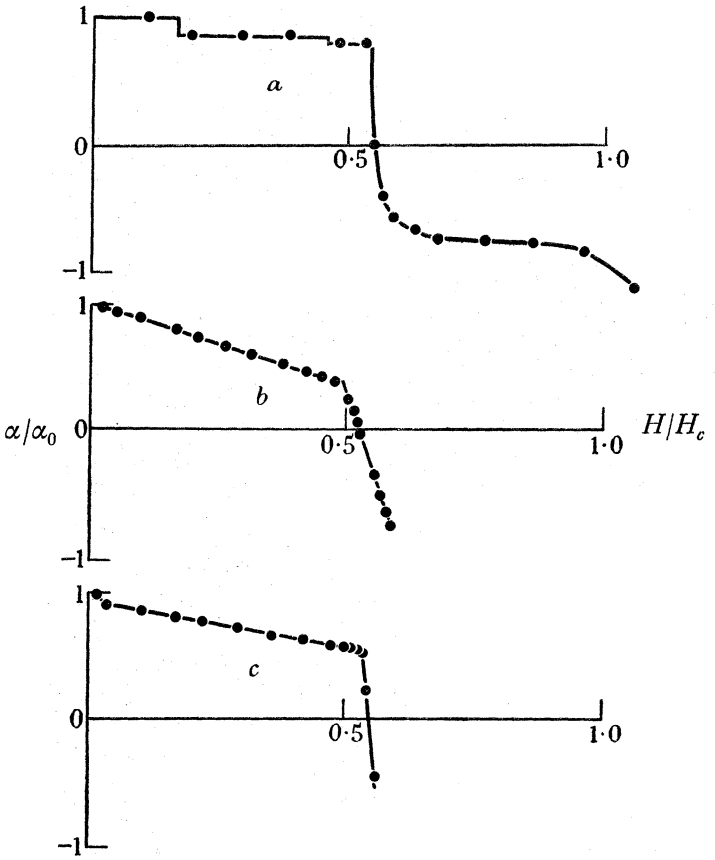

FIGURE 15. Removal of flux by reverse field: α_0 = trapped flux before application of reverse field; α = trapped flux after application of reverse field, H. (*a*) sp. 1 (pure Sn), $t = 0{\cdot}560$, $\alpha_0 = 0{\cdot}10$ %; (*b*) sp. 15 (2·46 % In), $t = 0{\cdot}988$, $\alpha_0 = 18$ %; (*c*) sp. 15 (2·46 % In), $t = 0{\cdot}600$, $\alpha_0 = 2{\cdot}5$ %.

the field at the surface, and helps to restore the normal state which is necessary for the elimination of trapped flux. After removing the small reverse field the magnetic moment of the rod was measured in the usual way, and the experiment was repeated with progressively larger reverse fields. The observations for two specimens (1 and 15), are shown in figure 15, from which it will be seen that the small amount of flux trapped in pure tin is very firmly bound, being almost completely fixed until the field reaches $\frac{1}{2}H_c$ and the intermediate state is set up. At that stage the reverse field enters easily and is partially trapped when the external field is removed. The larger amounts of trapped flux in impure tin are less firmly bound, but even so only about one-half has been eliminated by the time the reverse field reaches $\frac{1}{2}H_c$.

A related effect is the turning of the trapped flux by means of a field applied at right angles to the original direction. With sp. 15 it was found that a field of just less than $\frac{1}{2}H_c$ in the normal direction decreased the moment to about 0·7 of its original value and rotated it through 40°; again it is seen that the flux is not rigidly bound, but at the same time it is not completely mobile. With the pure tin specimen sp. 1, as might be expected from the small effect of a reverse field, a normal field produced no noticeable turning of the moment. This agrees with many similar earlier observations (Kamerlingh Onnes 1924; Love, Blunt & Alers 1949; Condon & Maxwell 1949).

4. Discussion of the experiments

The principal conclusions of the experimental investigation may be summarized as follows:

(*a*) For a smooth, homogeneous, superconducting rod the trapping of flux is a reproducible phenomenon in a given specimen.

(*b*) The variation with temperature of the proportion trapped follows a curve whose shape is characteristic of the concentration of indium, though the scale may vary considerably from specimen to specimen.

(*c*) For indium concentrations less than about 2·3 % the proportion trapped is zero at the transition temperature, rising steadily as the temperature is lowered.

(*d*) For indium concentrations greater than about 2·3 % the proportion trapped is of the order of 50 % just below the transition temperature, follows a curve similar to that for less impure specimens over an intermediate temperature range ($t \sim 0{\cdot}85$ to 0·6), and shows signs of rising rather steeply at the lowest temperatures.

(*e*) The trapped flux is rather firmly held, particularly in the purest specimens.

(*f*) Lattice defects influence the magnitude of the trapped flux but not its temperature variation, except that they may enhance the rapid rise in the proportion trapped by impure specimens at low temperatures and near the transition temperature.

As it will be a basic assumption in the analysis which follows these results that, apart from lattice defects, the specimens were homogeneous, it is as well to review the evidence on which this is based. Inhomogeneity of the indium concentration might arise from two causes, insufficient annealing and insolubility. From the evidence presented it seems a reasonable assumption that 20 days' annealing is sufficient to bring the specimen to a fairly final state, except for the effect observed after 124 days' annealing of sp. 14, which

is attributed to the elimination of lattice defects. Unfortunately, there appear to have been no measurements of the diffusivity of indium in tin, but if we assume that it is the same as that of tin in tin (Fensham 1950) we may deduce that the r.m.s. displacement of an indium atom during 20 days at 205° C is 3×10^{-2} cm, which should be adequate to smooth out the greater part of the concentration gradients which arise during crystallization. There may still be left slow variations along the length of the specimen, but these are unlikely to matter greatly; what is more important is homogeneity over distances comparable with the dimensions of normal and superconducting regions in the intermediate state, and as these are probably less than 10^{-2} cm it is likely that sufficient homogeneity was achieved. There is good evidence from these and related experiments that indium is soluble in tin to the extent of at least 3 %. If a second phase were formed during annealing it would be either superconducting or normal at 3·7° K, and its behaviour would probably depend on which it was. If normal, we should expect to observe something like the behaviour shown by sp. 22 (figure 7) containing insoluble zinc, and this is clearly not observed. On the other hand, if there were small superconducting inclusions they might behave quite differently so far as flux-trapping is concerned, but would be apparent in the resistance measurements. It has indeed sometimes been noted (e.g. Shoenberg 1952) that tin-indium alloys have a resistance which drops slowly between 4·2 and 3·7° K, but nothing of this sort was observed with the specimens used here. Moreover, Doidge (1954) has found that the residual resistance of tin-indium alloys varies nearly linearly with indium concentration up to 6 % at least. This may be contrasted with the results of Serin & Lohman (1953), who find that in alloys of tin with zinc, cadmium and lead there is a linear variation up to the limit of solubility, above which the residual resistance is independent of impurity concentration. It seems a safe assumption, then, that there was homogeneity of indium concentration in all the specimens.

5. Introduction to theoretical discussion

An attempt will now be made to explain the foregoing observations in terms of a somewhat idealized model of the superconducting state. It will not prove possible to account for the results in detail, if only because of the necessary inadequacy of the model occasioned by the imperfections of present knowledge. Moreover, it is clear that there may be a number of different mechanisms operating simultaneously, and there is insufficient evidence to enable an altogether convincing choice to be made of those which are of greatest importance. It appears, however, possible to account for most of the observations in a qualitative manner by means of the model chosen, and attention will be concentrated on the development of one particular interpretation which seems the most plausible, while no attempt will be made to hide the fact that it is based on rather speculative assumptions.

When the field is reduced from a value greater than H_c the specimen goes from the normal into the intermediate state, a complex mixture of normal and superconducting regions whose scale is determined, according to existing theories, by the dimensions of the specimen and the interphase surface energy. Continued reduction of the field causes the superconducting regions to expand at the expense of the normal regions, and as it is only the latter which can carry magnetic flux, the total amount of flux within the specimen

decreases with the applied field. For cylindrical specimens the normal regions would ideally be entirely eliminated at a field strength of $\frac{1}{2}H_c$, and there would be no trapped flux. But it may happen during the reduction of the field that neighbouring superconducting regions coalesce in such a way as to form a ring around a normal region. The flux within this region is now completely trapped, on account of the well-known flux-conserving property of a superconducting ring. It is possible that the normal region may be able to migrate to the edge of the specimen and in this way discharge its contained flux, but for the present we shall disregard this process. It will be examined in §8. Whether or not such migration is important, it is necessary in order to trap flux that coalescence of superconducting regions shall occur, since it is only in this way that the flux threading through normal regions can be denied an escape route to the outside of the specimen. The following discussion will be largely concerned with the mechanism of coalescence, and the way in which it may be affected by the addition of impurity. It will begin with an analysis of some elementary processes in terms of a simplified model, which will then be applied to the interpretation of the experimental results on flux-trapping.

6. Theory of idealized processes

6·1. *Model of a superconductor*

In this section is developed an idealized model of a superconductor which expresses, at least approximately, most of the relevant thermal and electrical properties which are known at present. In order that the probable validity of any results derived may be assessed, the principles on which it is based will be discussed in some detail, as well as the limitations in knowledge which give scope for considerable latitude in the choice of model.

The first, and basic, assumption to be made is that the superconducting state of a metal represents a partially ordered state of the electrons, such as can be described (as in the Bragg–Williams theory of order-disorder in alloys) by the introduction of a single order parameter ω. In the completely ordered state, the superconductor at 0° K, $\omega = 1$, and in the disordered, normal, state $\omega = 0$. The electronic contribution to the free energy per unit volume f will be taken to be a function of ω and of the reduced temperature t ($= T/T_c$, where T_c is the transition temperature), as well as of certain constants of the metal concerned. In the absence of a magnetic field ω_0, the equilibrium value of ω, is determined by the condition $(\partial f/\partial\omega)_t = 0$, and the corresponding value of $f(\omega_0, t)$ will be designated by $f_s(t)$. Now, to a good approximation, the specific heat of a superconductor is proportional to t^3, and the form which f_s must take is given very nearly by the equation

$$f_s(t) = -\beta(1+t^4), \tag{1}$$

where $\beta = H_0^2/8\pi$ and H_0 is the critical magnetic field at 0° K. Further, in the normal state the electronic specific heat is proportional to t, and when $t = 1$ there is a second-order transition between the two states. Hence

$$f(0, t) = f_n(t) = -2\beta t^2. \tag{2}$$

These two experimental conditions on the form of $f(\omega, t)$ leave still a wide choice, which may be narrowed somewhat by conditions which are less securely founded. According to

the two-fluid model of superconductivity, ω is to be interpreted as the fraction of electrons which are in the superconducting state. If this idea is combined with the theory of London & London (1935) it follows that the penetration depth λ varies as $\omega^{-\frac{1}{2}}$. Now experimentally it is found (Daunt, Miller, Pippard & Shoenberg 1948) that λ varies as $(1-t^4)^{-\frac{1}{2}}$, so that one is led to a third condition on the form of $f(\omega, t)$, that

$$\omega_0 = 1-t^4. \tag{3}$$

Even this condition leaves an infinite choice of $f(\omega, t)$ possible, and further narrowing down of its form must await experimental evidence on the non-equilibrium behaviour of superconductors, or on the shift in equilibrium induced by a magnetic field. To discuss in detail the possible ways of determining $f(\omega, t)$ more exactly would take us too far afield, and it will be necessary to assume for the present purpose either the form originally proposed by Gorter & Casimir (1934), which satisfies the conditions given above,

$$f(\omega, t) = -\beta\{\omega+2t^2\sqrt{(1-\omega)}\}, \tag{4}$$

or an equally plausible alternative form which Whitehead (1955) has recently pointed out, and which appears to agree better than (4) with his experimental results on colloidal particles,

$$f(\omega, t) = -\beta\left\{2t^2+2\omega\,\frac{1-t^2}{1+t^2}-\frac{\omega^2}{(1+t^2)^2}\right\}. \tag{5}$$

The expressions (4) and (5) are similar for values of t near unity, but are quite different at low temperatures. For example, when $t=0$, $f=-\beta\omega$ according to (4), while according to (5) $f=-\beta(2\omega-\omega^2)$. It should be remembered therefore that although the qualitative consequences of the model are unlikely to be seriously influenced by a different choice of $f(\omega, t)$, any quantitative results which may be deduced could readily be modified by a different, and equally plausible, choice. This point will be emphasized from time to time by comparing the results obtained in various calculations by the use of (4) and (5).*

Now that suitable expressions for the thermodynamical properties of the model have been chosen, its electromagnetic properties must be formulated, and for this purpose the theory of London & London (1935)will be adopted, in the form

$$\text{curl}\,(\Lambda\mathbf{J})+\mathbf{H}=0, \tag{6}$$

where $\Lambda=\Lambda_0/\omega$, and Λ_0 is a constant for a given superconductor. It will further be assumed that this equation is applicable even when, as in the present applications, ω varies from point to point. Equation (6) leads to the correct temperature variation of the penetration depth λ, which is easily shown from (6) and Maxwell's equations to be given by $\sqrt{(\Lambda/4\pi)}$. It has been found (Pippard 1953*a*) that the penetration depth is increased by the addition of indium to tin, so that Λ_0 must be assumed to depend on indium concentration, and not to be a function only of the number and effective mass of the conduction electrons as in the original London theory. This experimental result indeed casts grave doubts on the

* It may be noted that while the two terms in (4) are interpretable as separate contributions to f from the superconducting and normal electrons respectively (this is in fact how the equation was originally derived), no such simple interpretation of (5) is possible. It is rather dubious indeed whether (5) can be regarded as representing a two-fluid model, but this is no argument against its validity.

correctness of the London equation, but as no alternative formulation has been achieved which can be applied to the present problem it is necessary to adopt this rather unsatisfactory compromise in order to carry out any calculations.

Finally the model must incorporate the idea, which appears to be well substantiated by experiment, that ω may not vary too rapidly from point to point. According to the views of Ginsburg & Landau (1950) a rapid variation of ω results in an increase in the local energy density for quantum-mechanical reasons, and their theory has had some success in accounting for the properties of pure superconductors. On the other hand, it is based on the London theory and encounters difficulties in application to impure superconductors, in which the variations of ω appear to be limited not so much for energetic reasons as by scattering of electrons by impurities. It appears to be possible (Pippard 1953*b*) to represent the experimental results adequately by the supposition that ω may not change substantially in a distance less than ξ, the 'range of coherence', which is given by the equations

$$\frac{1}{\xi(0)} = \frac{1}{\xi_0} + \frac{1}{l}, \quad \text{at } 0^\circ \text{ K}, \tag{7}$$

ξ_0 being a constant ($\sim 10^{-4}$ cm for tin) and l the mean free path of conduction electrons in the normal metal, and

$$\xi(t) = \xi(0)/\sqrt{(1-t^4)}. \tag{8}$$

Thus although the magnitude of ξ_0, which dominates equation (7) in pure tin, may be controlled by processes of the type envisaged by Ginsburg & Landau, in impure tin for which l may be considerably less than ξ_0, it seems that the scattering mechanism is the more important, and the calculations which follow will be based on a crude application of the latter hypothesis.

In order to incorporate the idea of coherence consistently, it will be assumed that only such variations of ω with position are possible as can be made up by the superposition of units of a given type, and for convenience the unit will be chosen to have a radial variation of ω of the form

$$\omega(r) = \frac{2\xi\omega'}{r} \tanh\left(\frac{r}{2\xi}\right) \operatorname{sech}^2\left(\frac{r}{2\xi}\right). \tag{9}$$

This has a maximum value of $\omega\,(=\omega')$ at the centre, and at distances large compared with ξ the variation of ω is as $\exp(-r/\xi)/r$, so that ξ may be interpreted as analogous to a mean free path. In accordance with this probably over-simplified model the formation of a superconducting nucleus in an otherwise normal matrix is pictured as the growth from zero of the parameter ω' in (9), ξ meanwhile remaining constant. Similarly a uniform superconductor in equilibrium may be described as a dense array of units having this form, with ω' so adjusted that the value of ω resulting from the superposition of all the units is equal to ω_0 everywhere. And the most rapid variation of ω from ω_0 to zero at a phase boundary is achieved by filling the space to one side of the boundary with such a dense array of units while leaving the other side completely empty. A straightforward integration shows that if the negative half-space $x<0$ is so filled, the positive half-space being empty, the variation of ω at the resulting phase-boundary follows the law

$$\omega(x) = \tfrac{1}{2}\omega_0\{1-\tanh(x/(2\xi))\}. \tag{10}$$

We are now in a position to use this model to calculate certain relevant properties of a superconductor, and the way in which they depend on impurity, i.e. on the value of ξ.

6·2. *Interphase surface energy*

The origin of the surface energy between superconducting and normal phases has been discussed independently by Ginsburg & Landau (1950) and by Pippard (1951), who agree in ascribing it to the diffuse nature of the phase boundary, and we may use the model developed above and the form (10) for the phase boundary to calculate the surface energy explicitly. The phase boundary (normal to the x-axis) is in equilibrium only if the applied field, parallel to it, is equal to H_c; and if at any point x the local field strength is $H(x)$, the local density of the Gibbs function may be written in the form

$$g(x) = f(\omega, t) - H_c H(x)/(8\pi),$$

so that for a portion of the material lying between $\pm\infty$ on the x-axis, and having unit cross-sectional area, the Gibbs function may be written

$$G = \int_{-\infty}^{\infty} f\,\mathrm{d}x - \frac{H_c}{8\pi}\int_{-\infty}^{\infty} H\,\mathrm{d}x. \tag{11}$$

Now define a 'configurational boundary' as the plane $x = X_C$, such that

$$\int_{-\infty}^{\infty} f\,\mathrm{d}x = \int_{-\infty}^{X_c} f_s\,\mathrm{d}x + \int_{X_c}^{\infty} f_n\,\mathrm{d}x,$$

where f_s and f_n are given by (1) and (2), and a 'magnetic boundary' as the plane $x = X_M$, such that

$$\int_{-\infty}^{\infty} H\,\mathrm{d}x = \int_{X_M}^{\infty} H_c\,\mathrm{d}x.$$

Then from (11), remembering that $f_n - f_s = H_c^2/(8\pi)$, we have that

$$G = \int_{-\infty}^{\infty} f_s\,\mathrm{d}x + (X_M - X_C)\,H_c^2/(8\pi). \tag{12}$$

Now if there were no surface energy at the interface, and this was localized at the plane $x = x'$, the Gibbs function would be given by the equation

$$G' = \int_{-\infty}^{x'} g_s\,\mathrm{d}x + \int_{x'}^{\infty} g_n\,\mathrm{d}x = \int_{-\infty}^{\infty} f_s\,\mathrm{d}x,$$

since when the applied field is H_c, $g_s = g_n = f_s$. Hence we may interpret the second term in (12) as the surface energy per unit area, α_{ns}, and write for the surface energy parameter Δ, which is defined as $8\pi\alpha_{ns}/H_c^2$,

$$\Delta = X_M - X_C.$$

The calculation of Δ is thus reduced to a calculation of the positions of the configurational and magnetic boundaries. To find X_C we substitute (10) in (4) and integrate over all x; it turns out that to a close approximation $X_C = t\xi$. If (5) were used instead of (4) we should find $X_C = \xi$, independent of t. There is thus a significant quantitative difference between the results obtained with the two forms of $f(\omega, t)$.

The calculation of X_M is slightly more involved, as it requires the numerical solution of the differential equation for the field. By use of Maxwell's equation curl $\mathbf{H} = 4\pi\mathbf{J}$, and introduction of the vector potential $\mathbf{A}$, (6) takes the form

$$\frac{\mathrm{d}^2 A}{\mathrm{d}x^2} = \frac{4\pi}{\Lambda_0}\,\omega A = \frac{2\pi\omega_0}{\Lambda_0}\left\{1 - \tanh\left(\frac{x}{2\xi}\right)\right\} A, \tag{13}$$

from (10), where A is written for A_y, the only non-vanishing component of **A**. The solution of this equation numerically, subject to the condition $\lim_{x\to-\infty}(A) = 0$, is straightforward. As x increases the right-hand side of (13) tends to zero, so that $\mathrm{d}A/\mathrm{d}x$ tends to a constant value. It is easy to see that X_M is given by the limit, as $x_0 \to \infty$, of the expression $\left\{x_0 - \left[A \Big/ \left(\frac{\mathrm{d}A}{\mathrm{d}x}\right)\right]_{x=x_0}\right\}$, and in this way the magnetic boundary is determined. If ξ is much greater than λ $(=\sqrt{\{\Lambda_0/(4\pi\omega_0)\}})$, the value of X_M and hence of Δ is found to depend critically on the exact form chosen for the variation of $\omega(x)$, so that no reliance can be placed on this method of calculating Δ for pure tin. But when ξ and λ are comparable the form of $\omega(x)$ does not matter so much, and, moreover, it is reasonable to suppose that (10) is not too bad an approximation if coherence is limited by impurity scattering of the electrons. We may therefore expect this analysis to give a fair indication of the value of ξ at which $X_M = X_C$ and the surface energy vanishes (for smaller values of ξ, $\Delta < 0$). The result of the computations is shown in figure 16; we shall defer further discussion until the end of § 6·3.

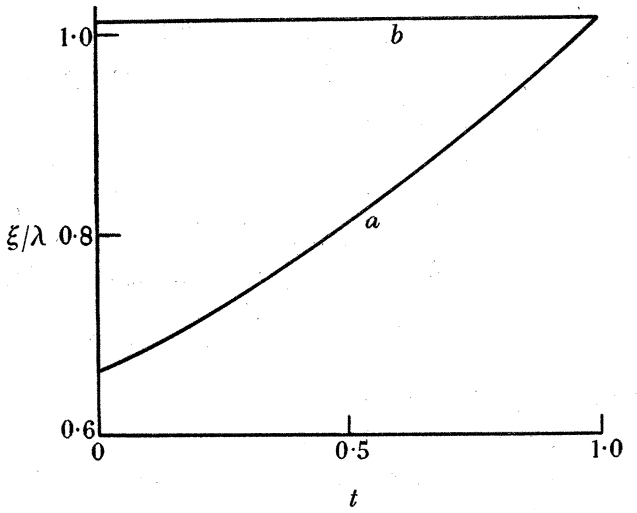

FIGURE 16. Temperature variation of value of ξ/λ at which $\Delta = 0$: (*a*) according to Gorter–Casimir, equation (4); (*b*) according to Whitehead, equation (5).

6·3. *Spontaneous nucleation*

If the field applied to a wholly normal specimen is reduced from a value greater than H_c, the superconducting phase becomes thermodynamically stable as soon as $H < H_c$. But the mechanism whereby the phase change occurs, involving the growth of a superconducting nucleus in the normal matrix, may not be energetically possible on account of a potential barrier separating the pure normal and superconducting phases. If we consider the growth of a nucleus of the form (9) in an applied field H we may calculate the dependence of the Gibbs function on ω'. Initially, as ω' rises from zero, G changes through two causes; as a result of the increase of ω', magnetic efforts being ignored, G is decreased, while the diamagnetism of the nucleus leads to an increase of G. If the nucleus is large in comparison with the penetration depth the latter magnetic effect dominates, with the net result that G increases, and formation of the nucleus is prohibited. As H is reduced the

magnetic term falls, until at some critical value H' the potential barrier vanishes and spontaneous nucleation becomes possible. If H' is greater than H_c, as may occur when very small nuclei are permitted, the nuclei will be unable to expand until H is reduced below H_c; but if $H'<H_c$, the nuclei, once formed, can expand to convert the whole specimen into the superconducting state. The problem of calculating the critical field H' for nucleation is very similar to one already treated (Pippard 1952), the supercooling of colloidal particles, and it is sufficient to state the result of applying the same method to a nucleus of the form (9):

$$\frac{H'}{H_c} = \frac{\sqrt{6}}{\pi}\frac{\lambda}{\xi}(1+t^2)^{\frac{1}{2}} \quad \text{from (4)},$$

or

$$\frac{2\sqrt{3}}{\pi}\frac{\lambda}{\xi} \quad \text{from (5)}.$$

Thus the condition for nucleation to be just possible when $H = H_c$ takes the form

$$\xi/\lambda = \{6(1+t^2)\}^{\frac{1}{2}}/\pi \quad \text{from (4)},$$

or

$$2\sqrt{3}/\pi \quad \text{from (5)}.$$

In the model adopted the temperature variation of ξ is the same as that of λ, so that according to Whitehead's expression (5) for $f(\omega, t)$ nucleation becomes possible at the critical field for a certain impurity content which is independent of temperature. If this impurity concentration is exceeded, spontaneous nucleation becomes possible in fields greater than H_c. It is noteworthy that Doidge (1954) has found evidence for such spontaneous nucleation when the indium concentration in tin exceeds 2·5 %, and that the critical concentration is temperature-independent. This evidence supports Whitehead's expression (5) rather than that of Gorter & Casimir (4), but it is hardly sufficient to allow a convincing decision between the two.

Comparison of the result just obtained with the result of our calculation of Δ (figure 16) shows that spontaneous nucleation at the critical field should occur at slightly lower impurity concentrations than that required for Δ to vanish, the condition for the former being that $\xi = 1{\cdot}10\lambda$ and for the latter that $\xi = 1{\cdot}01\lambda$. Since we suppose that at 0° K $\xi(0)$ is very nearly equal to l, the mean free path, and there is evidence (Pippard 1953*a*) that at rather high impurity concentrations the penetration depth at 0° K varies as $l^{-\frac{1}{3}}$, it is to be expected that $\xi/\lambda \propto l^{\frac{2}{3}} \propto \rho_{\text{res.}}^{-\frac{2}{3}}$, where $\rho_{\text{res.}}$ is the residual resistivity of the specimen. And since $\rho_{\text{res.}}$ is proportional to the indium concentration in tin, we expect the critical concentration of indium for the vanishing of Δ to be 1·06 times that for spontaneous nucleation at the critical field. The difference between these two is just enough to make it clear that the two effects need not occur at exactly the same concentration, but the figure 1·06 cannot be regarded as having any great significance in view of the approximation of the model.

The indium concentration at which λ and ξ are equal may be estimated from data on the variation of $\lambda(0)$ with l (Pippard 1953*a*). On account of the high anisotropy of the anomalous skin effect in tin (which is used to determine l), there is some doubt about the exact figures quoted, but it is probable that the indium concentration at which $\lambda = \xi$ lies between 2·2 and 2·8 %. This agrees very satisfactorily with Doidge's (1954) result that at 2·5 % spontaneous nucleation begins.

6·4. *Coalescence of superconducting regions*

The calculation of the conditions under which two neighbouring superconducting regions can coalesce is much more difficult than the two problems just tackled, and no solution has been found. We can, however, establish qualitative results by simple arguments, since the process involved is very similar to that of spontaneous nucleation. Consider two superconducting regions separated by a thin normal lamina having plane parallel boundaries, and carrying a field H_c. These boundaries will not be sharp, but ω will vary in a direction perpendicular to the boundaries in a manner shown schematically in figure 17. Herein lies the first difficulty: if ω varies at a boundary according to (10) it cannot drop to zero between two such boundaries, however far apart they may be. Since in practice there is no doubt about the existence of a laminated intermediate state we must infer either that the exponential tail of (10) can be cut off, as indicated by the broken line of figure 17*a* or that when ω drops to a certain low value, $\omega_{\text{min.}}$, as in figure 17*b*, the

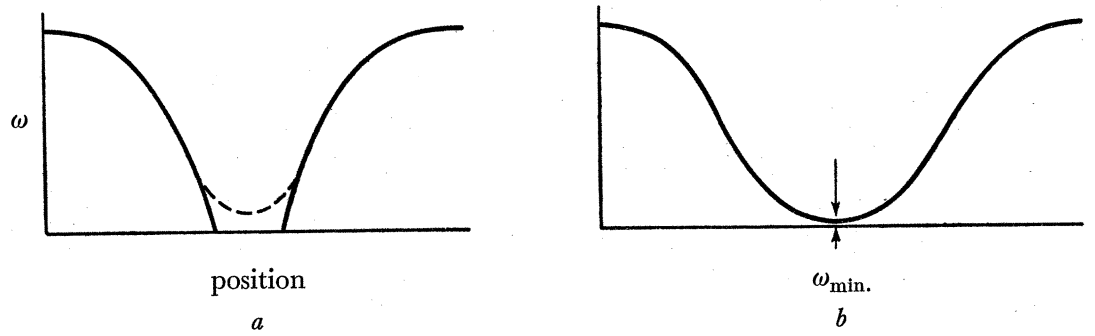

FIGURE 17. Schematic variation of ω across gap separating two superconducting regions; (*a*) and (*b*) represent different models of the interface.

material no longer exhibits superconducting properties. There seems to be no serious objection to the latter view, which was suggested for other reasons by Doidge (1954). The consequences of these two views in the present problem are somewhat different. According to the first we may suppose that as the two superconducting regions draw together there comes a time when, the separation of the effective phase boundaries being of the order of ξ, no closer approach is possible without the elimination of the intervening normal lamina, as indicated by the broken line in figure 17*a*. The question whether this process of coalescence is energetically possible involves the same notions as were used in discussing spontaneous nucleation. On the one hand the Gibbs function is lowered by the general increase in the value of ω; on the other hand, as soon as ω rises from zero in the intervening normal region there must begin the expulsion of flux, with a consequent increase in the Gibbs function. The relative magnitude of these two effects determines whether the process will begin. Even in the simplest case, the bridging of the gap between two plane phase boundaries by the interposition of a single unit of the form (9), the calculation of the magnetic contribution involves the solution of equation (6) under very awkward conditions, which has not been achieved. We have therefore to rely on intuitive arguments. It seems most probable, from the similarity of this process to spontaneous nucleation, that the condition for the occurrence of coalescence is very nearly the same as for nucleation in a field H_c, and

it may be expected that coalescence is not energetically possible unless ξ is of the order of, or less than, λ. Just as for nucleation, once the first bridge has formed it may be able to expand and give rise to an extended junction of the superconducting regions. In particular, if by some chance such a bridge forms in a less impure material for which $\xi > \lambda$, the positive surface energy at the interface will aid the consolidation of the junction; if, on the other hand, $\xi < \lambda$, the initial process of coalescence will be achieved without hindrance, but the negative surface energy will prevent consolidation until the field is reduced below H_c.

Turning to the second alternative, illustrated in figure 17*b*, we may suppose that the two phase boundaries approach until the value of ω between them reaches $\omega_{\text{min.}}$; thereafter any further approach involves the expulsion of flux from the whole intervening space. If ω_0 is only slightly larger than $\omega_{\text{min.}}$, as at a temperature just below T_c, there is not a great difference between the two cases, and we should expect nearly the same condition for coalescence to be possible, i.e. $\xi \sim \lambda$. But if ω_0 is considerably greater than $\omega_{\text{min.}}$, as at lower temperatures, the phase boundaries may need to be separated by several times ξ before an effectively normal region is created between them. This has the effect of increasing the magnetic barrier to coalescence. Thus if the second alternative is correct we may predict that when enough impurity is added so that ξ is less than λ, coalescence of neighbouring superconducting regions will be possible at temperatures just below the transition temperature, but still impossible at lower temperatures. Further addition of impurity should have the effect of diminishing the critical value of t below which coalescence is forbidden.

The conclusion of this argument, then, is that there should be no coalescence, and hence no flux-trapping, in specimens containing less than about 2·5 % indium (which is the critical concentration found by Doidge for spontaneous nucleation in a field H_c), but that for greater concentrations the possibility of coalescence may allow flux-trapping to occur readily, though the expected temperature variation of α, the fraction trapped, is highly dependent upon the choice of model. The fact that for concentrations less than 2·5 % indium, a certain, if small, amount of trapping occurs in practice indicates that there are some relevant factors neglected in our idealized model, and we must now proceed to analyze the observed phenomena in terms of a somewhat more realistic picture.

7. Mechanisms of flux-trapping

7·1. *The nature of the intermediate state*

Let us attempt to follow the transition of a rod in a transverse field from the normal state in a field greater than H_c to the almost completely superconducting state in a field less than $\frac{1}{2}H_c$. As the field applied to a pure specimen is lowered the transition usually does not start at H_c, but is delayed owing to the difficulty of creating superconducting nuclei. If spontaneous nucleation, as discussed above, were the only mechanism available a very high degree of supercooling would be expected in pure tin, but, as Faber (1952) has shown, nucleation occurs more readily at certain points in the rod; these are interpreted as the sites of suitable lattice defects, in the neighbourhood of which ξ is smaller than in the perfect crystal. Once such a nucleus has formed it may readily grow into a rod parallel to the applied field, and if the field is less than H_c may also expand sideways. Now the expansion of such a rod involves the displacement of magnetic flux, and the eddy currents

so induced in the surrounding normal metal increase the field at the phase boundary and limit the rate of expansion (Pippard 1950; Lifshitz 1950; Faber 1954). It is not difficult to see that under these conditions the phase boundary is unstable; the symmetrical expansion of a cylindrical nucleus is slower than the growth of laminar offshoots, provided the latter are not so thin as to be held back from expansion by the interphase surface tension. Again, these laminae, as they grow into the normal regions, tend to expand sideways, but the same instability prevails and in their turn they send out offshoots, so that eventually the whole specimen becomes filled with a maze of interlinking normal and superconducting laminae, which may owe their origin to only a few primary nucleations. If coalescence of superconducting regions is forbidden, as discussed above, there will be no normal regions completely surrounded by superconducting material, and there will remain normal paths along which flux may escape from the specimen. As the field continues to decrease, the superconducting regions may expand or proliferate further, and more flux will be ejected as the normal regions shrink.

To take the argument beyond this point we must consider the dimensions of the normal and superconducting regions. No calculations have been made for the complex structure suggested above, but Landau (1937) has considered an idealized model of plane laminae, and his ideas have been advanced further by Lifshitz & Sharvin (1951). The latter authors have discussed Landau's (1943) second model of the intermediate state, in which the normal laminae are considered to undergo multiple branching along the direction of the applied field. As they have shown that for most purposes the original unbranched model is energetically more favourable, we shall base our discussion on this treatment. When the proportion of normal material, η, is close to unity the thickness of superconducting laminae, a_s, which minimizes the Gibbs function is $(\pi \Delta d/\ln 2)^{\frac{1}{2}}$, where d is the thickness of the specimen in the direction of the field; the theory has been worked out only for a plate subjected to a normal field, and we shall take a cylinder of radius r to have an effective thickness of $r\sqrt{\pi}$, so that

$$\lim_{\eta \to 1} (a_s) = \pi^{\frac{3}{4}} (\Delta r/\ln 2)^{\frac{1}{2}}.$$

The variation of a_s with η, according to the calculations of Lifshitz & Sharvin, is shown in figure 18, together with the corresponding variation of a_n, the equilibrium thickness of the normal laminae, and the sum $a_n + a_s$. It will be seen that as η decreases to about $\frac{1}{2}$, $a_n + a_s$ decreases while a_s remains nearly constant. This may be interpreted in our model of the intermediate state as a continued proliferation of superconducting laminae, gradually filling more and more of the intervening normal space. Further decrease of η, however, involves an increase of $a_n + a_s$, that is, a reduction in the total number of laminae. Doubtless in a perfect crystal such a rearrangement would be possible and the equilibrium configuration would always prevail; but the Gibbs function has a very flat minimum corresponding to the values of a_s and a_n shown in figure 18, so that it is likely that even small inhomogeneities (such as the flaws postulated for nucleation, where Δ varies with position) will prevent the free movement of phase boundaries. We shall therefore suppose that as η is lowered $a_n + a_s$ falls to its minimum value and remains there. With further reduction of the field the superconducting laminae simply expand into their surrounding normal regions. Eventually, when η is small, the normal regions form an intricate mesh, still however connected to the exterior of the specimen if no coalescence has occurred. This

view of the process is not entirely unsubstantiated. Meshkovsky & Shalnikov (1947) have published diagrams of the laminar arrangement which indicate a remarkable immobility of the laminae as the field is changed, and Désirant & Shoenberg (1948) have found that in small cylinders the magnetization curve is not completely reversible; it is clear then that the true equilibrium configuration is not necessarily reached.

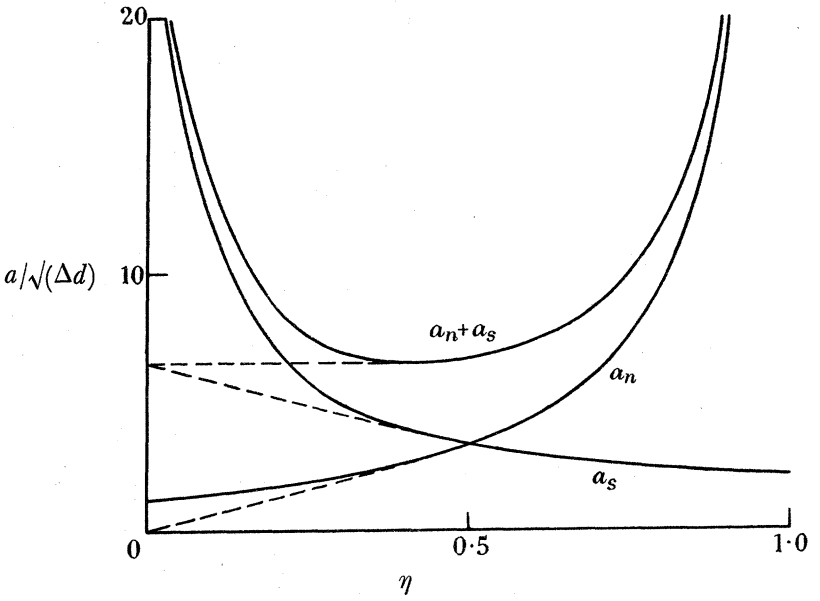

FIGURE 18. Scale of superconducting and normal regions in intermediate state; the full lines are those calculated by Lifshitz & Sharvin (1951), the broken lines what is here assumed to obtain in a decreasing field.

According to Lifshitz & Sharvin the minimum value of $a_n+a_s \doteq 9(r\Delta)^{\frac{1}{2}}$, so that when η is small the width of the normal laminae should be given approximately by the expression

$$a_n = 9\eta(r\Delta)^{\frac{1}{2}}. \tag{14}$$

Now this treatment of the intermediate state is based on the hypothesis of a sharply defined phase boundary, and we should not expect it to be satisfactory if it predicts a value of a_n which is much smaller than the coherence range ξ. In fact, if we put $a_n = \xi = 10^{-4}$ cm, $r = 0{\cdot}2$ cm, $\Delta = 2\times 10^{-5}$ cm (Faber 1954), values appropriate to a specimen of pure tin at 0° K, we find the corresponding value of η to be about $\frac{1}{2}\%$. Further reduction of the applied field, which ideally should lead to the elimination of all normal material, will be unable to reduce η much further, unless the phase boundaries are to be shortened by rearrangement of the domains. The observation that in pure tin the trapped flux may amount to only 0·1 % strongly suggests that, as expected, coalescence is very hard to achieve, and it is quite likely that the normal channels persist at their minimal width, and carrying a field less than H_c, until they finally vanish when the external field is reduced considerably below $\frac{1}{2}H_c$.

7·2. *The influence of flaws*

The variability of the amount of flux trapped, from one specimen to another of the same purity, and the increase in trapping occasioned by addition of small amounts of indium, suggest that this is not the whole story, and that the factor really responsible for

coalescence and flux-trapping is the existence of flaws, of the type found necessary by Faber (1952) in order to account for nucleation of the supercooled phase. As we have seen there is a great similarity between nucleation and coalescence, so that in tin containing about 1% indium, the absence of any observed supercooling implies that there are sites at which coalescence can occur fairly readily. According to this view the difference between one specimen and another lies mainly in the density of the flaws, and some support is provided by the otherwise rather odd behaviour of the polycrystalline sp. 19 (see figure 5) which at temperatures below $0{\cdot}9T_c$ was similar to that of a single-crystal specimen, except that the amount of flux trapped was much less than for the best single-crystal specimen. It appears from the figure that the effectiveness of grain boundaries as traps diminishes rapidly as the temperature is lowered, and that once the temperature is low enough for their effect to be negligible there are only a few flaws available for trapping, since most of the lattice defects have been, as it were, swept away by the movement of grain boundaries during annealing.

A simple model of a flawed specimen may be imagined to contain a number of regions in which ξ and Δ decrease to very low values at the centre of the flaw. Many of these flaws will serve as nucleation centres and become incorporated in the superconducting phase at an early stage, but some may be presumed to remain in the normal phase. When a phase boundary approaches such a flaw it will be attracted by the gradient of Δ towards the centre, and if there are two superconducting laminae separated by the flaw they will be drawn together at this point and eventually will coalesce there, if the flaw is sufficiently potent to allow coalescence in a field equal to H_c. Let us suppose that the effective size of each flaw is a' (assumed for simplicity to be the same for all flaws), so that coalescence begins when $a_n = a'$, i.e. according to (14) when $\eta = \eta' = \frac{1}{9}a'(r\Delta)^{-\frac{1}{2}}$. Once coalescence has started, the positive surface energy in the bulk of the material will tend to increase the area of coalescence and eliminate phase boundaries as far as possible, unless the gradient of Δ near the flaw is so steep as to counteract this tendency. Under these circumstances the consolidation of the junction will be delayed until the field is reduced further. If there are only a few regions of coalescence a considerable part of the normal material will still be connected to the edge of the specimen and the flux it contains will be swept out of the specimen. But where multiple coalescence occurs normal regions will be isolated from the edge and hence there will be flux-trapping. Although α, the amount of flux finally trapped, will depend on the density and potency of the flaws, and vary considerably between specimens, we may probably assume that for a given specimen it is proportional to η'. Moreover, the effective size of the flaws is not likely to vary greatly with temperature, so that for a given specimen α should vary as $\Delta^{-\frac{1}{2}}$. In figure 19, $\Delta^{-\frac{1}{2}}$, taken from Faber's (1954) values for pure tin, is plotted against t, as well as the experimental curve for α in sp. 5 (1% indium). The agreement is particularly good near the transition temperature; the discrepancy at lower temperatures should not be regarded as significant in view of the approximate nature of the theory and the absence of information on whether Δ hasthe same temperature-dependence in alloys as in pure tin, as our simple model predicts.

Qualitatively, one can see from this model why α should have a tendency to increase with the concentration of indium. Even if the density of flaws were the same in samples of

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different purity the fact that Δ decreases with addition of indium is sufficient to account roughly for the observations. It would involve too much *ad hoc* refinement of the theory to explain in detail why the shape of the curves for α varies with purity, so that as the indium content increases the fall of α to zero at the transition temperature gets progressively sharper, and we shall not attempt this here. It seems probable, however, that it is related to the presence of a wide range of flaw sizes and we shall indicate how this may affect the issue by consideration of the difference between the curves obtained for pure tin and for tin containing 1% indium. As we have seen, the latter is fairly well explicable in terms of the temperature variation of Δ, but in pure tin the value of α falls much more gradually to zero (figure 8), and the theory developed above is clearly incorrect here. The modification needed is indicated by Faber's (1952) study of supercooling in tin. He has shown that when pure it may supercool considerably, and that flaws are necessary to nucleate the superconducting phase. If there are no flaws which are capable of allowing nucleation in a field as great as H_c it is probable that there are none which can aid

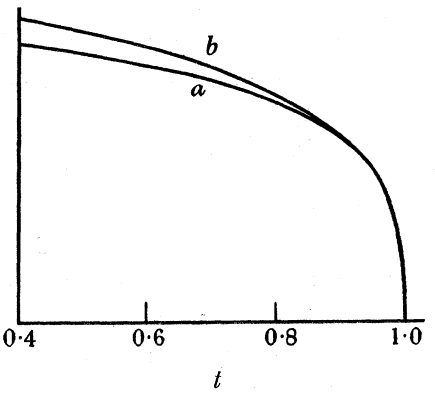

FIGURE 19. Temperature variation of (*a*) $\Delta^{-\frac{1}{2}}$, (*b*) α for sp. 5. The scale of the ordinates is arbitrary.

coalescence, since we believe the two processes to be very similar. Moreover, the most potent flaws are likely to be used in the initial nucleation of the intermediate state and hence to be unavailable for coalescence. It may therefore be supposed that coalescence does not begin until the applied field is reduced below $\frac{1}{2}H_c$, and the normal regions have attained their minimal thickness which, being roughly proportional to ξ, will be greater at higher temperatures. The small flaws which are still available will thus be less able to effect coalescence at higher temperatures, and the applied field must be reduced to a lower fraction of H_c than at lower temperatures; in consequence the proportion of trapped flux will be smaller. Such a mechanism would account qualitatively for the different shapes of the curves for pure and impure specimens, but it is no more than a tentative suggestion, unconfirmed by other evidence. Nevertheless, the extent to which a qualitative description can be given of the behaviour of pure tin and alloys containing not too much indium supports the idea that coalescence is only achieved through the help of flaws in the neighbourhood of which the range of coherence is reduced to a value comparable with the penetration depth.

7·3. *Flux-trapping in the more impure specimens*

When we turn to the specimens containing more than about 2·5 % indium we expect an abrupt change in behaviour corresponding to the possibility of spontaneous nucleation and coalescence even in the unflawed regions of the specimen, and something of the sort is indeed observed. The fact that the rise of α to a high value as t approaches unity occurs at the same indium concentration as that at which Doidge (1954) has observed the persistence of superconducting material in fields exceeding H_c, is strong evidence that the two effects are closely related. We may suppose that, in such an impure specimen, as the field is reduced from a high value the material is at first entirely normal; then as the critical field H' for nucleation is passed a large number of thin superconducting filaments, about ξ in radius, are formed, but cannot enlarge substantially until the field falls below H_c. Once this has happened the filaments may expand sideways and coalesce with their neighbours if there is no energy barrier to the latter process. According to the model represented in figure 17*a*, if spontaneous nucleation is possible, so also is coalescence in a field H_c. Thus as soon as the field falls below H_c the material should become filled with an intricate superconducting filigree, a very efficient trap for flux. It may be noted that there should be no tendency for the surface energy to consolidate the superconducting regions into domains, since under such conditions that coalescence becomes possible Δ is very small or even negative.

This cannot be the complete explanation, however, for according to Doidge (1954) spontaneous nucleation sets in at about the same impurity content for all temperatures. Accordingly, if the flux trap suggested above can work at one temperature it should work at all. The experimental results, on the other hand, while clearly indicating the appearance of a new mechanism at temperatures near T_c, show that at lower temperatures this new mechanism is inoperative and the process of flux-trapping is the same as in less impure specimens. We are led to the conclusion, then, that coalescence without the aid of flaws is energetically possible in these specimens only at the higher temperatures, and this conclusion accords with the alternative model which is represented in figure 17*b*; the phase boundaries are kept apart by the requirement that ω shall not exceed some critical value $\omega_{\min.}$ midway between them, and the separation, relative to ξ, increases as the temperature is lowered on account of the increase of ω_0. No attempt has been made to give this tentative explanation a quantitative basis, but so far as it goes it seems to support the view that ω does not drop steeply to zero at a phase boundary, but exhibits something like an exponential tail. This is the conclusion reached by Doidge (1954) from his study of the details of spontaneous nucleation.

Finally, attention should be drawn to the way in which α begins to rise steeply at the lowest temperatures when the indium concentration exceeds 2·5 % (see figure 11). This may be related to the surface energy parameter Δ becoming negative, but we are unable to offer any detailed explanation here.

7·4. *Flux-trapping in imperfect specimens*

A rapid rise in α at the transition temperature is exhibited by imperfect specimens, as shown in figures 5, 6 and 13. It is reasonable that a specimen such as sp. 14 (figure 13),

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containing nearly enough indium for spontaneous nucleation to occur, should be highly sensitive to such residual effects of slip as are left after 34 days' annealing, and if it contains a high density of small flaws it may well simulate the behaviour of a more impure specimen. But the effect of grain boundaries, as shown in figure 5, is puzzling. The crystallites in sp. 19 were several millimetres in size, and just below T_c were able to cause the trapping of as much as 8 to 10 % of the flux, yet at lower temperatures they had apparently only a negligible influence. We might suppose that a grain boundary would behave as a large flaw, but its effectiveness seems to be confined to a narrow temperature region near T_c. Perhaps, as Faber (1952) has suggested, the effective flaws consist of a network of lattice defects extended in three dimensions, and the substantially two-dimensional flaw which constitutes a grain boundary may not provide a sufficiently large volume over which ξ is greatly reduced. If this is so the reason for the high values of α near T_c must be sought in some other property of the polycrystal. Near a boundary the lattice distortion may give rise to a local variation in T_c and H_c, but more probably it is the anisotropic thermal expansion of tin which throws a polycrystalline specimen into a highly strained state on cooling and leads to local variations of the critical parameters. At temperatures near T_c any slight variations will be more important than at lower temperatures, and the specimen may then be significantly inhomogeneous and exhibit 'sponge' behaviour (Mendelssohn 1935). A similar explanation may be advanced for the trapping of flux in unannealed or partially annealed specimens (figure 6), which again may be inhomogeneous to an extent which matters only in the vicinity of T_c. In this connexion it is worth noting that the transitions to the superconducting state in a small field ($\sim$1 G) were noticeably broader for those specimens (7 and 19) which were known to be inhomogeneous than for any other. For example, the transition of sp. 19 was about $\frac{1}{100}°$ wide, while that of sp. 20, containing 2·6 % indium, was not more than $\frac{2}{1000}°$ wide.

This last observation has another and more important significance—it implies that the reason for the rise of α to a high value near T_c in specimens containing more than 2·5% indium is not to be found in macroscopic inhomogeneities such as are present in the imperfect specimens (7 and 19). We have thus additional support for the view implied in the discussion of § 7·3 that the behaviour of the most impure specimens reflects a fundamental property, rather than an undetected type of inhomogeneity.

8. Migration of flux

Our discussion has been based entirely on the assumption that once a normal region is wholly surrounded by superconducting material the flux it contains is permanently trapped. It is by no means obvious that this should be so, as Faber (1954) has pointed out. For if the phase boundaries can move in such a way as to shift the normal region to the side of the specimen the magnetic energy will be reduced; the process of migration is thus energetically favourable. The mechanism by which migration may be accomplished is readily understood from figure 20, representing the field lines at the end of a normal channel. On account of the curvature of the lines as they leave the specimen, and the fact that curl $\mathbf{B} = 0$ in the normal material, it is clear that the field is greater at the corners than in the centre of the channel, and in particular the field at A is greater than at B, since the

curvature is sharper for the former. Thus the channel as shown cannot be stable, but must tend to move towards the right by the conversion of the region round A into normal material and of the region round B into superconducting material. Further, if in the body of the channel $H = H_c$ all around the phase boundary, the channel must remain straight, and therefore the tendency is for the channel to migrate as a whole towards the edge. It might therefore be argued that the discussion of coalescence is irrelevant, since even if it leads to the isolation of extended normal regions migration will eventually eliminate the trapped flux.

There is evidence, however, that this view is too naïve. For example, the application of a reverse field, as shown in figure 20, should greatly aid the migration of flux, since it enhances the field at A and reduces it at B. Nevertheless, as can be seen from figure 15, the trapped flux is remarkably stable against the influence of a reverse field. Certainly some of the flux is eliminated, so that migration is not entirely negligible, but the effect

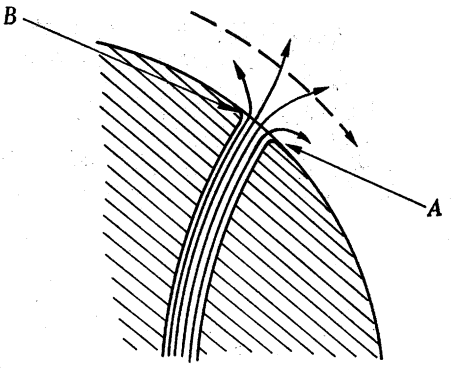

FIGURE 20. Flux lines at end of normal channel. The superconducting regions are shaded; the broken curve shows the direction of a reverse field aiding migration.

does not seem to be important as a basic mechanism. The fact that the curves in figure 15 are nearly linear in the range of field from 0 to $\frac{1}{2}H_c$ suggests that they may safely be extrapolated backwards to $-\frac{1}{2}H_c$ to determine how much flux is trapped within the specimen when the field has been reduced from a large value to $\frac{1}{2}H_c$. If this is a valid extrapolation the flux left in the specimen when the field is reduced to zero is at least 60 % of that trapped at $\frac{1}{2}H_c$. Since usually the measured values of α are less than 5 % it follows that the greater part of the flux escapes while the specimen is in the intermediate state, when the magnetic forces aiding migration are very weak. It might of course be argued that a great deal of flux is indeed trapped and migrates readily, and that figure 15 represents the behaviour of only those channels which are hindered by lattice defects. But this would raise a serious difficulty of understanding why the curves b and c for sp. 15 are so alike, when α is so different from the two, being 18 and 2·5 % respectively. In addition, one can see qualitatively that a marked difference in trapping behaviour should occur, as observed, when spontaneous coalescence becomes possible at indium concentration around 2·5 %; it is not so clear that any marked change in migration should occur then. We conclude, therefore, that the part played by migration in eliminating

flux is rather small, and that the fundamental process involved is the difficulty of spontaneous coalescence except at flaws and in very impure material.

This conclusion raises a further problem: why is migration so unimportant? A simple answer would be that the normal inclusions dispose themselves around lattice defects, at which a gradient of critical field or surface energy opposes the magnetic force drawing them towards the edge. But other effects may operate, even in a nearly flawless sample. If, as suggested, the intermediate state is a tangle of normal and superconducting regions, the trapped flux, when the external field is removed, will lie in a labyrinth of normal material, perhaps as sketched in figure 21. At a point such as X the magnetic force will draw the normal inclusion towards the edge, but this force will be opposed (so long as the difficulty of coalescence prevents a part of the inclusion being detached from the rest) by the surface energy which seeks to contract the area of the phase boundaries. There may thus well be a stable balance of forces keeping the flux securely locked in the specimen.

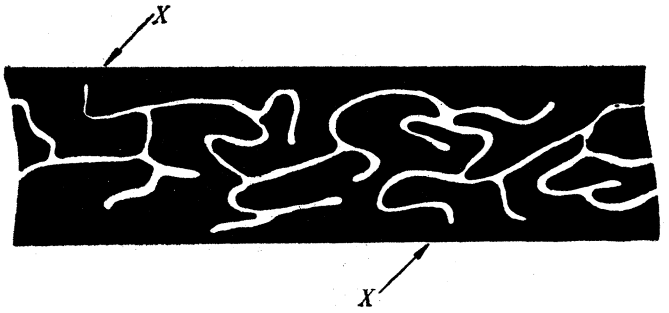

FIGURE 21. Schematic cross-section of rod, perpendicular to applied field, to illustrate presumed network of normal channels. The dark regions are superconducting.

9. CONCLUSION

We have sought here to account for the experimental evidence on flux-trapping by means of a model of the superconducting state in which the hypothesis is made that coherence plays a fundamental role in inhibiting the coalescence of neighbouring superconducting domains. Although nothing like a complete explanation has been given the degree of success attained encourages the belief that the model is not too far from the truth. In particular, the fulfilment of its prediction that a radical change in behaviour can be effected by addition of sufficient impurity is strong evidence in support of the suggested dependence of coherence range on the mean free path of the conduction electrons.

I should like to express my indebtedness to Dr T. E. Faber for many discussions and critical comments, and to Mr P. R. Doidge and Mr C. S. Whitehead for permission to make use of their unpublished work.

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